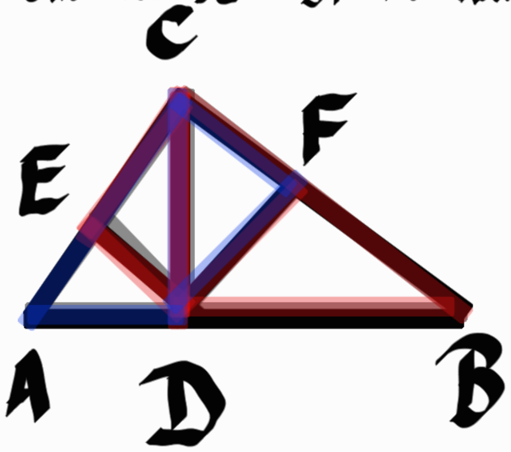
6. Iz nožišta D visine iz vrha C pravog kuta pravokutnog trokuta ABC spuštene su okomice $\overline{DE}$ i $\overline{DF}$ na katete trokuta. Dokažite da je $|CD|^3=|AB|\cdot|DE|\cdot|DF|$.



znamo: $$P=\frac{|BC|\cdot|AC|}{2}=\frac{|AB|\cdot|CD|}{2}$$

$$\Rightarrow |BC|\cdot|AC|=|AB|\cdot|CD|$$

$\square$: uočimo: $\triangle DEC \sim \triangle CDB$

$$\Rightarrow \frac{|DE|}{|CD|}=\frac{|CD|}{|BC|}$$

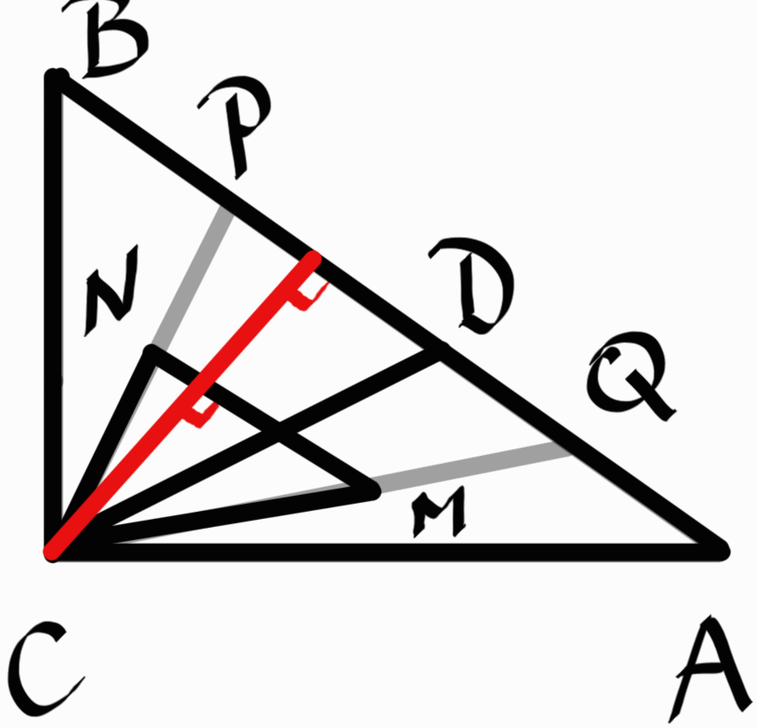
$$\Rightarrow |CD|^2=|DE|\cdot|BC|$$

$\square$: $\triangle ADC \sim \triangle CFD$

$$\Rightarrow |CD|^2=|DF|\cdot|AC|$$

$\square+\square$: $$|CD|^4=|AC|\cdot|BC|\cdot|DE|\cdot|DF| = |AB|\cdot|CD|\cdot|DE|\cdot|DF|$$

7. Dan je pravokutan trokut ABC kojem su duljine kateta $|AC|=7$, $|BC|=4$. Na hipotenuzi je odabrana točka D. Neka je M težište trokuta ADC i N težište trokuta BCD. Odredite površinu trokuta CMN.



P = polovište $\overline{BD}$

Q = polovište $\overline{AD}$

$$|MN| = \frac{1}{2}\cdot\underbrace{\frac{1}{2}|BD|}_{\text{srednjica}} + \frac{1}{2}\cdot\underbrace{\frac{1}{2}|AD|}_{\text{srednjica}}$$

$$= \frac{1}{4}|AB| = \frac{1}{2}|PQ|$$

↳ koef. sličnosti

$$P(CMN) = \frac{1}{4}P(CPQ)$$

$$1.\ d(C,MN) = \frac{1}{2}d(C,PQ) = \frac{1}{2}d(C,AB)$$

Površina velikog trokuta:

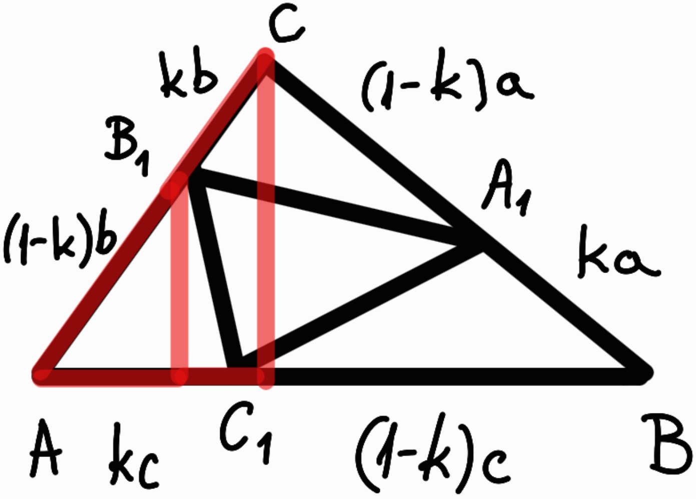
$$\frac{7\cdot 4}{2}=\frac{|AB|\cdot d(C,AB)}{2}$$

$$d(C,AB)=\frac{28}{|AB|}=\frac{28}{\sqrt{16+49}}=\frac{28}{\sqrt{65}}\Rightarrow d(C,MN)=\frac{14}{\sqrt{65}}$$

$$|MN|=\frac{1}{4}AB=\frac{\sqrt{65}}{4}$$

$$\Rightarrow P(CNM)=\frac{|MN|\cdot d(C,MN)}{2}=\frac{1}{2}\cdot\frac{\sqrt{65}}{4}\cdot\frac{14}{\sqrt{65}}=\frac{7}{4}$$

8. Na stranicama $\overline{BC}, \overline{CA}, \overline{AB}$ trokuta ABC dane su točke A_1, B_1, C_1 takve da $|BA_1|=k|BC|$, $|CB_1|=k|CA|$, $|AC_1|=k|AB|$ za neki $k\in\langle 0,1\rangle$. Označimo s P, S, P_1, P_2, P_3 redom površine trokuta ABC, $A_1B_1C_1$, AB_1C_1, A_1BC_1, A_1B_1C. Dokažite $P_1=P_2=P_3$. Ako je $S=k\cdot P$, odredite k.



$\triangle$: trokuti su slični s koef. $1-k$, zato

$$P_1 = \frac{1}{2}|AC_1| \cdot v_{B_1} = \frac{1}{2} kc \cdot (1-k) v_c$$

$$= k(1-k)P$$

Analogno $P_2 = P_3 = k(1-k)P$.

$$P = S + P_1 + P_2 + P_3$$

$$\Rightarrow S = P - 3k(1-k)P$$

po pretpostavci

$$= (3k^2 - 3k + 1)P = k \cdot P$$

$$\Rightarrow 3k^2 - 4k + 1 = 0$$

$$\boxed{k_1 = \frac{1}{3}}, \cancel{k_2 = 1}$$

$k \in \langle 0, 1 \rangle$